The Hanging Toss

Consider the displacement y(t):
(Think of it as a vertically tossed ball on a very strange planet!)

$$y(t) = 200t/3 - 5t^2 + (t^3)/6 - (t^4)/480$$

Take three derivatives and plot y(t) , v(t) , a(t) and j(t) from t=0 to 40.
Observe that the thing seems to "hang" momentarily at y=333.33:

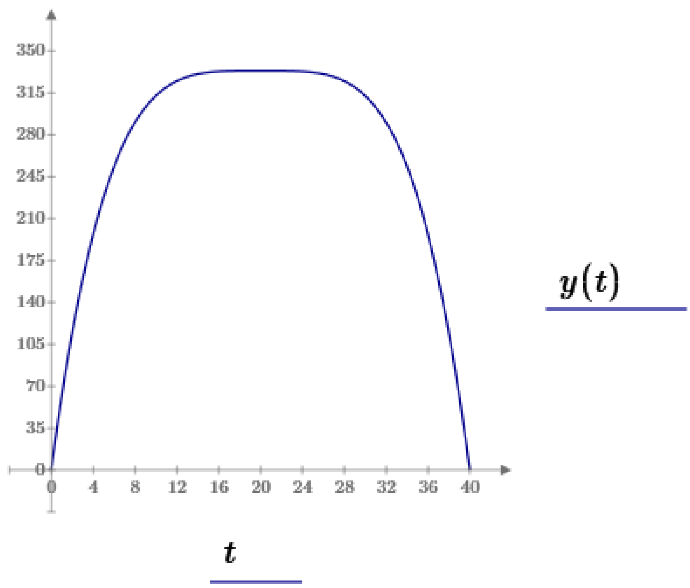
At t=20 the object has reached the top of its path. It then turns around and falls.
But at t=20, the velocity, the acceleration and the jerk are ALL zero!

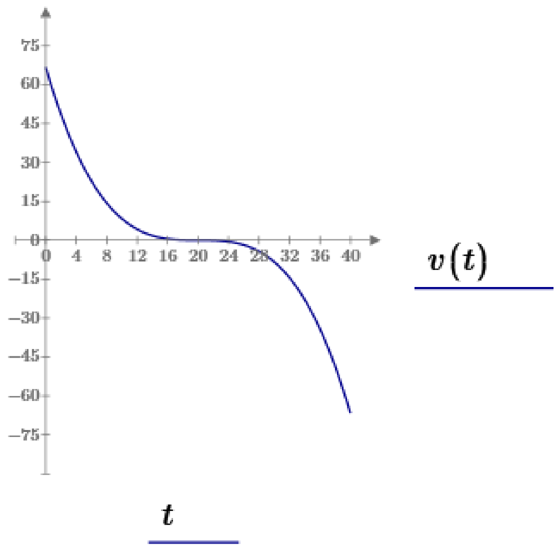
It only keeps going (turns around) because the derivative of its jerk is non-zero (-1/20). ALL other derivatives of y(t) are zero at t = 20.
This should disabuse one of (sometimes spoken, sometimes implied) arguments that the acceleration (and/or jerk) cannot be zero at a turnaround point simply because the velocity is obviously changing.

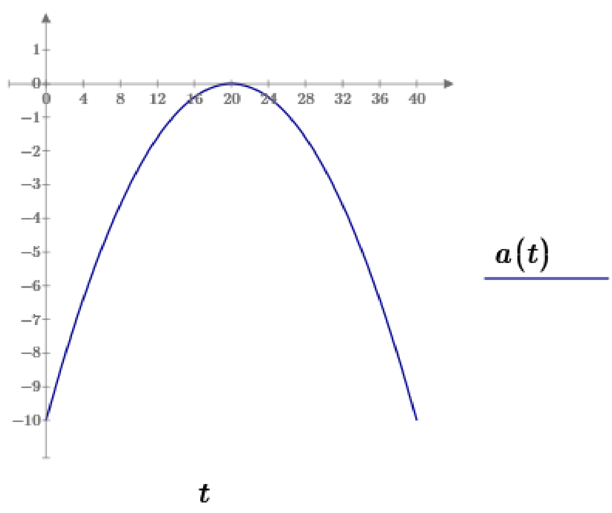
Even in the real Earth bound toss, there is nothing “special” about the turnaround event. Indeed its location in spacetime, and its very existence, are frame dependent. In the frame of a rising helicopter it will be re-located in spacetime (a different event in the ball’s history), and may even disappear (no turnaround event at all).

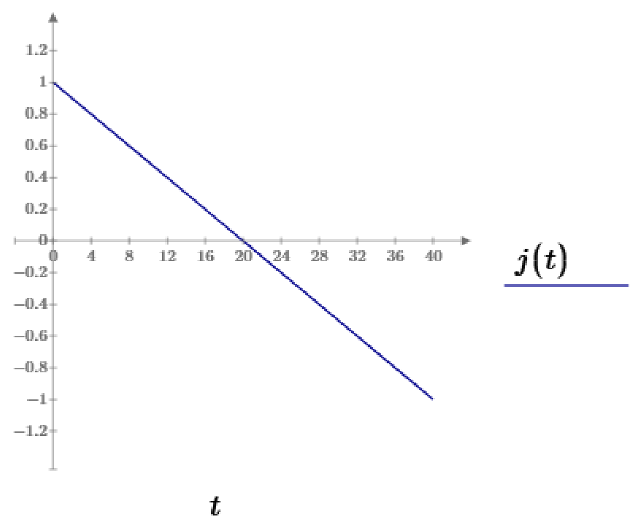
You can concoct for yourself more bizarre (but possible) motions in which the velocity keeps changing even though (at some time) it and all of its derivatives but the Nth one are zero, N being as large as you please.

Next page for the plots ==>


$y(t)$
t


$v(t)$
t


$a(t)$
t


$j(t)$
t