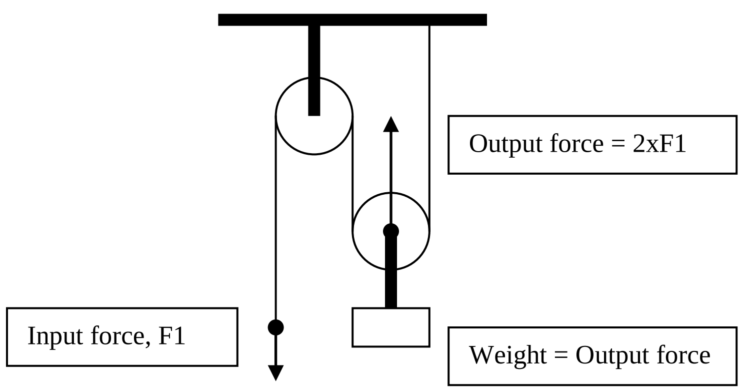
The magic of the rigid body

Robert J Sciamanda

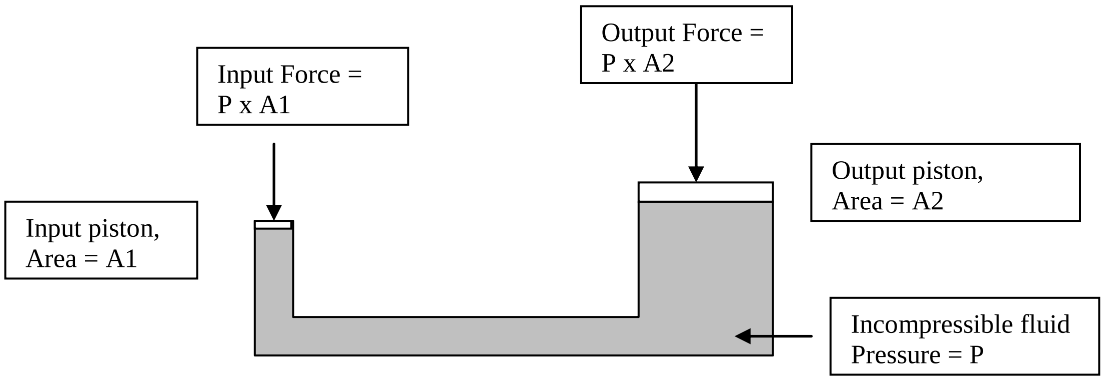
The inter-molecular mechanism by which a rigid body lever transmits a magnified force across a fulcrum is investigated. A two dimensional solid state lattice is simulated using damped springs to provide the inter-molecular forces. The time development of the simulation quantitatively exposes the details of the macroscopically hidden mysteries of the force multiplication. The model can also be used to examine the behavior of the solid state lattice in other circumstances, such as a cantilevered beam. The commercial program “Interactive Physics” is used to construct the simulation and generate graphical displays of the development in time of the inter-molecular forces.

I. INTRODUCTION

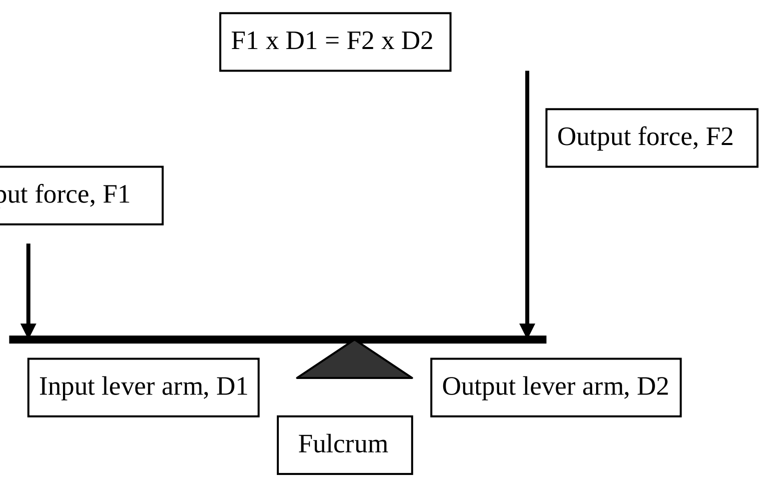
The use of a simple machine to multiply a force is one of the most frequent applications of physics in everyday life. Examples of simple machines in common use are the lever, the pulley system and the hydraulic actuator. In our teaching, we typically appeal to the energy conservation principle (in the form "work in equals work out") to quantitatively evaluate the ideal mechanical advantage of these machines. Good pedagogy will delve more deeply and seek to identify the underlying mechanism whereby the multiplied output force is directly produced. For the typical pulley system we can point to the tension developed in the rope (constant throughout the rope, in the absence of friction) by the input force and then simply count the number of rope segments which operate together to produce the output force .



For the hydraulic actuator we can explain that the input force operating on the area of the input piston establishes a hydraulic pressure which is constant throughout the confined liquid volume (Pascal's law). This common pressure then produces the output force by operating on the area of the output piston.



But when we consider the lever, there is no macroscopically obvious and transparent mechanical mechanism that quantitatively exposes the direct causes of the multiplied output force. The origins of this output force are shrouded in the hidden, internal, rigid body properties of the lever arm which connects the input and output forces across the fulcrum.

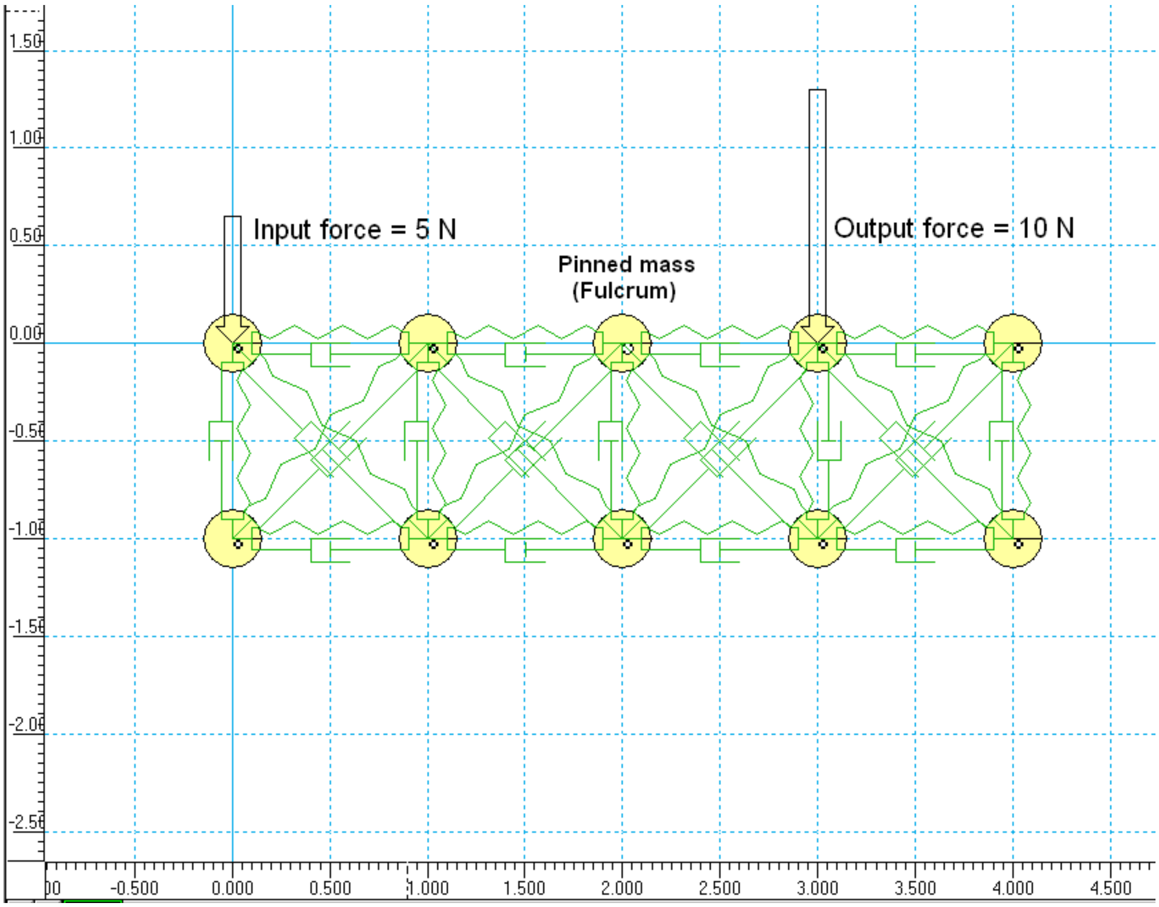


II. THE MYSTERY

Macroscopically there is a certain “magic” in the behavior of the rigid body, which is well illustrated in the lever: “Just what is the direct mechanism whereby the input force is multiplied and transmitted across the fulcrum to produce the output force?” Because we see and use the rigid body so often and so easily, we have become jaded and no longer wonder at its “magical” properties. If we had never seen a rigid body, I doubt that we would have imagined the possibility of such properties. Of course, the ideal, undeformable. perfectly rigid body is a practical impossibility, but this does not obviate the mystery. Very real, even though somewhat deformable, imperfectly rigid solids are usable as practical lever arms and really do transmit and multiply real forces with no loss of mystery to the curious observer.

III. GOING INSIDE THE RIGID BODY

This discussion seeks to expose and display the internal mechanism of the rigid body “magic” by using the atomic theory model of a solid as a system of molecules held together in a lattice structure by strong inter-molecular forces. We construct a crude, two dimensional mathematical model of the lattice, consisting of ten circular masses, each connected to its nearest neighbors by damped springs.



Input force = 5 N
Output force = 10 N
Pinned mass
(Fulcrum)
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-0.500
0.000
0.500
1.000
1.500
2.000
2.500
3.000
3.500
4.000
4.500

We add (simulations of) a fulcrum and input and output forces and consider the set of differential equations of motion for the ten masses. The solution of this equation set should then directly illustrate the time development of the internal rigid body forces and, in particular, the production of the multiplied output force.

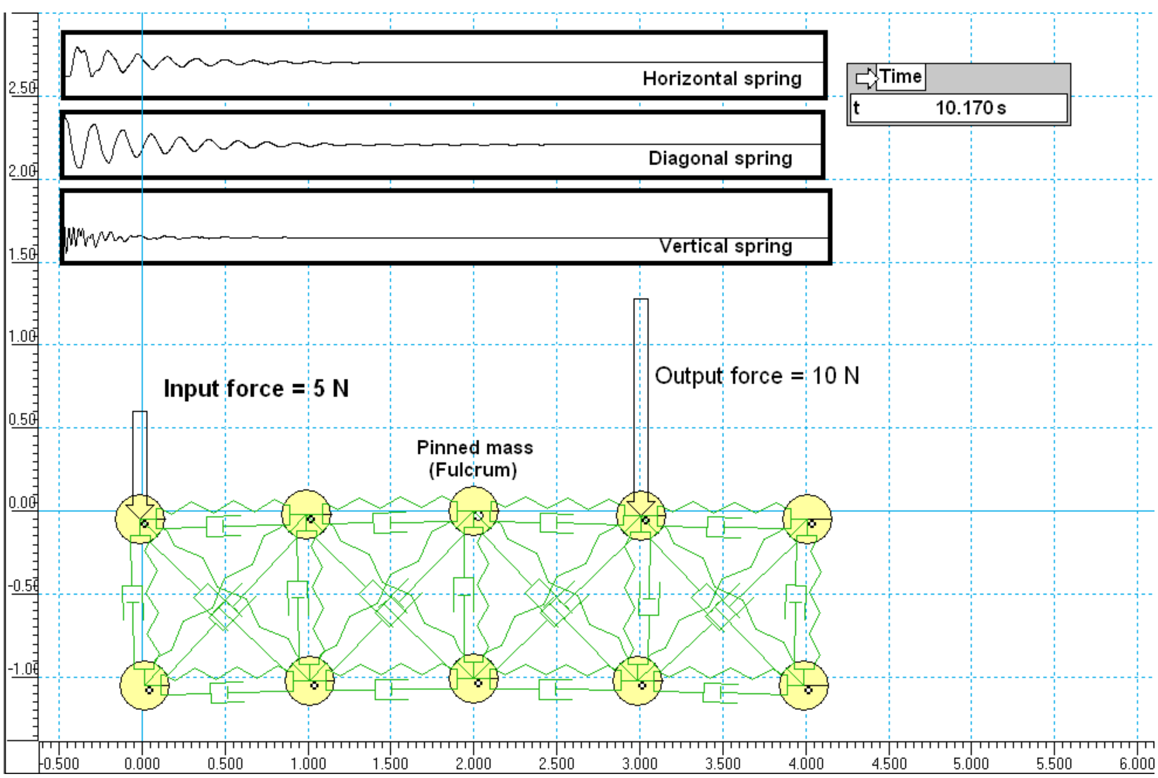
Even for this crude, two dimensional system of only ten masses and twenty one springs, this is a formidable calculational task. Fortunately, a commercial computer program exists that can be implemented to perform this task, and can even generate quantitative, time evolving graphical illustrations of the model! This program is the well known program “Interactive Physics”, sold by Design Simulation Technologies All of the simulations used in this paper were generated by Interactive Physics.

IV. THE SIMULATION DETAILS

Each molecule is simulated by a 1 kg circular mass of 0.15 meter radius. Each inter-molecular force is simulated by a spring/damper combination with a spring constant of 5000 N/m and a damping constant of 5 N/(m/sec). A downward gravitational field of g = 10 N/kg is imposed on the entire system, giving each circular mass a weight of 10 N. Input and output forces of five and ten Newtons are imposed, seeking a mechanical advantage of two. All of these values were rather arbitrarily chosen by trial and error so as to produce a well behaved system with small deformations, whose oscillations would dampen out within about ten seconds of simulated system time.

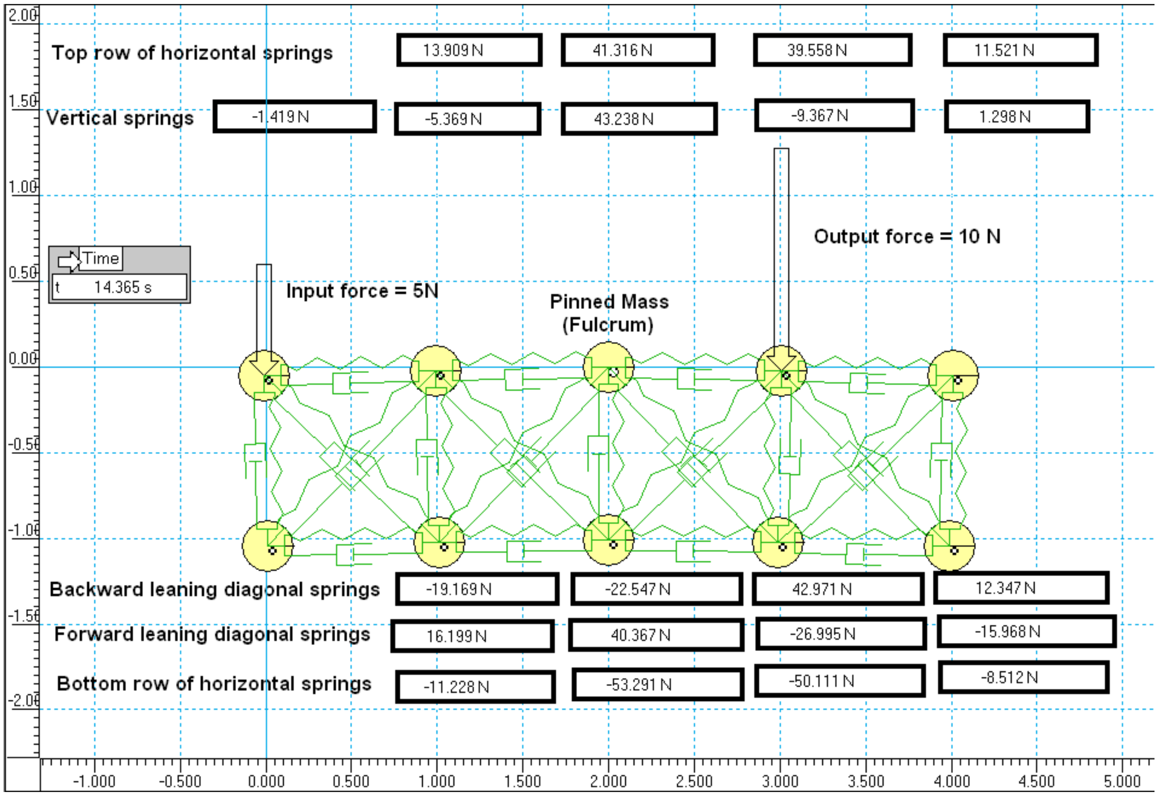
V. RUNNING THE SIMULATION

Once the simulation is started, one can watch the damped oscillations of the masses and springs on the Interactive Physics output screen. Cartesian graphs of these oscillations can also be generated. All of this action can be saved to a file and re-played as a movie. This figure shows the resulting state of the system after about 10 seconds of simulated system time. Graphs of the damped oscillations of the three springs attached to the upper, leftmost mass have been recorded and are also shown.



VI. THE FINAL SYSTEM STATE

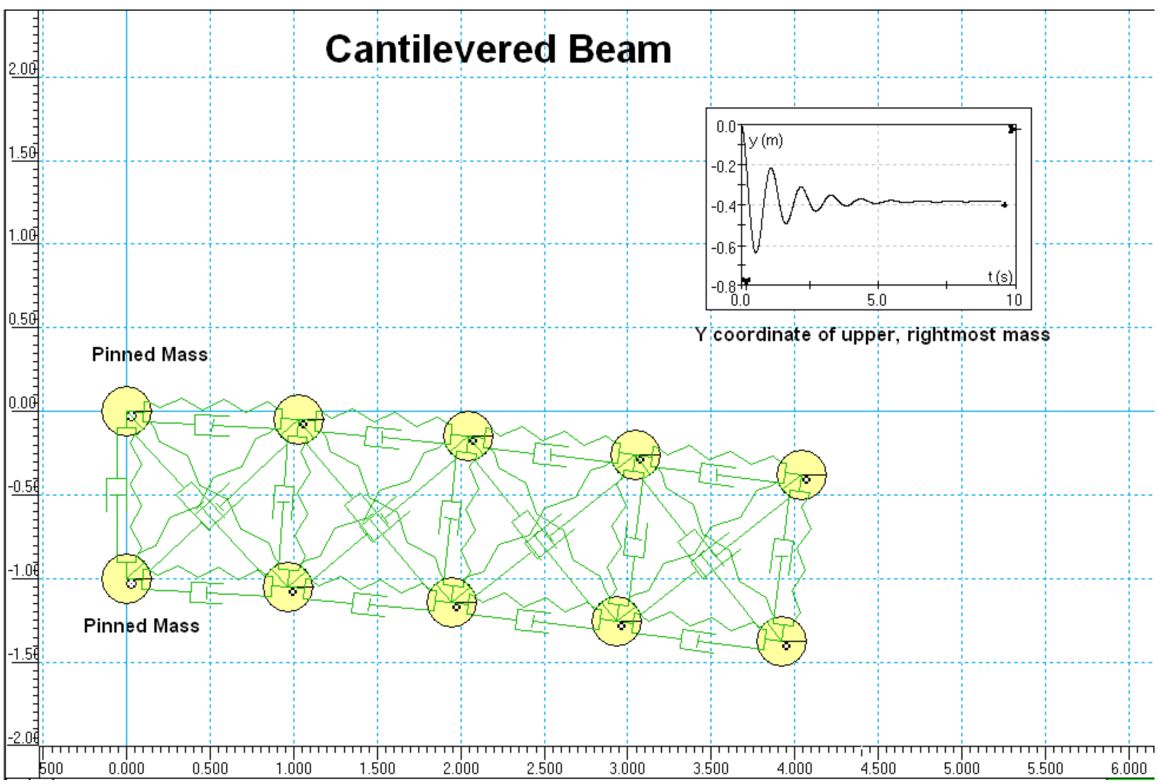
For a detailed quantitative analysis of the final equilibrium system state, The next figure displays the tension in each of the ten springs after about ten seconds of simulation time (Interactive Physics displays a compression as a negative tension).

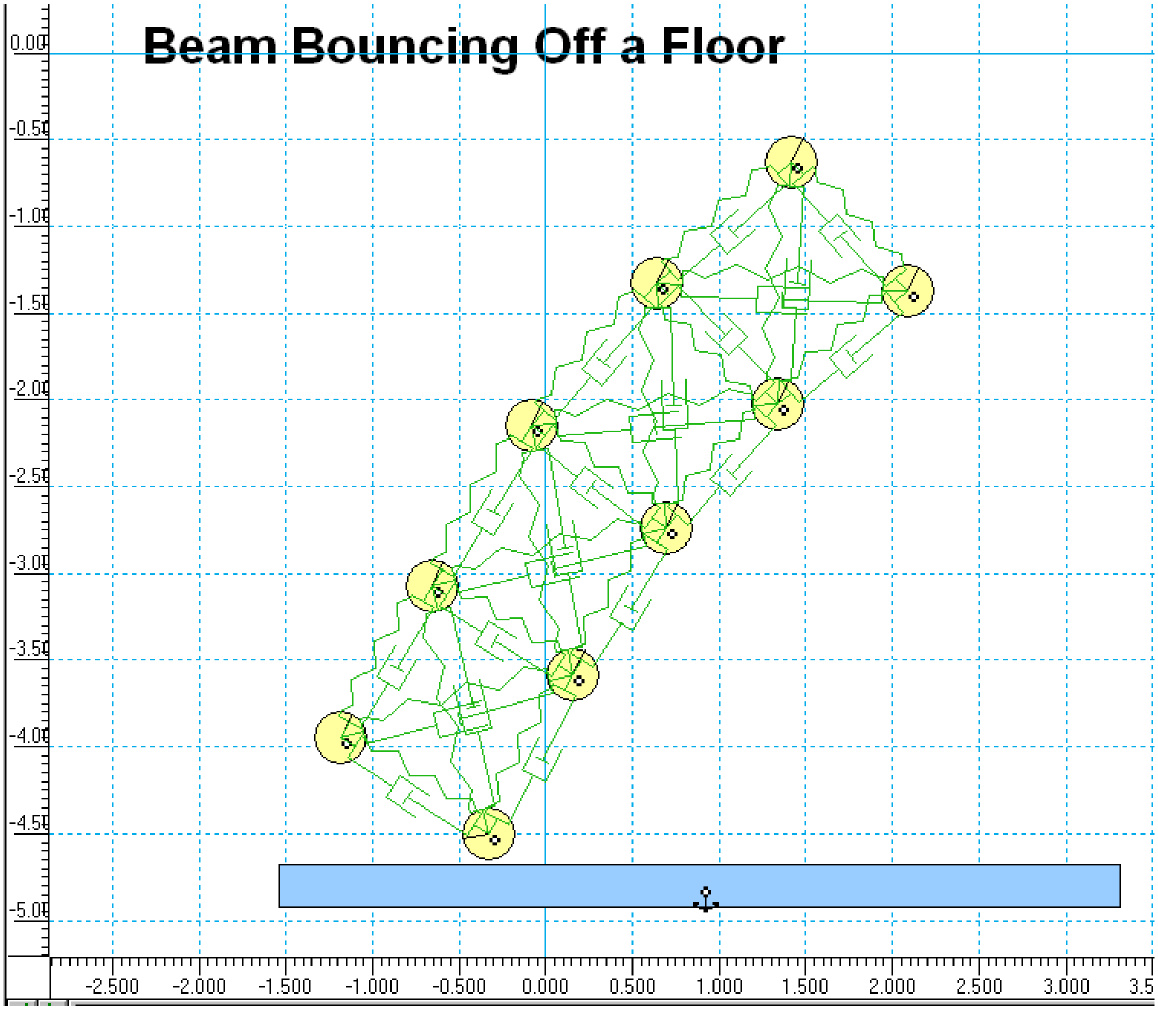


From this data, the student can observe the quantitative effect of each spring and evaluate the final, equilibrium forces on each mass. In particular, the direct causes of the input and output internal forces become apparent. The "mystery" is now heightened to a wondrous sense of awe in watching how these many forces conspire to obey the requirements of energy conservation and torque equality, producing the output force as the required multiplication of the input force.

VII. FURTHER STUDENT STUDIES

Students are delightfully entertained and enlightened by observing the behavior of this rigid body model in other situations, of their own invention. The next two figures show examples of a cantilevered beam, and a falling beam bouncing off the floor. The time behavior of each of these situations, along with quantitative, time evolving evaluations of all forces and displacements are easily generated by launching these programs within Interactive Physics.





This discussion is adapted from an oral presentation given by the author at a meeting of the Western Pennsylvania Section of the American Association of Physics Teachers at Washington & Jefferson College, Washington, Pennsylvania on Oct 10, 1998.