Getting from Boyle, Charles and Gay-Lussac to PV=CT

These three empirical studies show that a fixed quantity of gas approaches the following behavior as the Pressure is reduced toward zero. (The temperature T is measured from absolute zero) ==>

(1) Boyle: $P*V = f(T)$

(2) Charles: $V/T = g(P)$

(3) Gay-Lussac: $P/T = h(V)$

This low pressure behavior is called “Ideal Gas” behavior.

The experiments which generated these equations explored the behavior of two of the three variables (P, V and T) while the third variable is held constant. As written above, these equations cover the more general gas behavior by asserting that the right hand side of each of the three above equations can depend on only one of the three variables (T, P and V, respectively). The above equations tentatively quantify this fact by introducing the three unknown functions of only one variable f(T), g(P) and h(V).

We seek to combine the information in the above three equations to produce a completely general equation which will be valid even if all three of the variables (P, V and T) vary (but always staying in the low pressure, “Ideal Gas” domain). This means we seek to explicitly evaluate these three unknown functions (finding one of them will suffice to evaluate them all).

The mathematical logic is simple, but profound (the reasoning involved will recur in your more advanced Physics studies) What follows is a beautiful example of the awesome power of mathematical reasoning - clear and simple to any contemplative mind:

Solve (1) for P and substitute that result into (3). This yields

$f(T) / (V*T) = h(V)$, or

$$V*h(V) = f(T)/T$$

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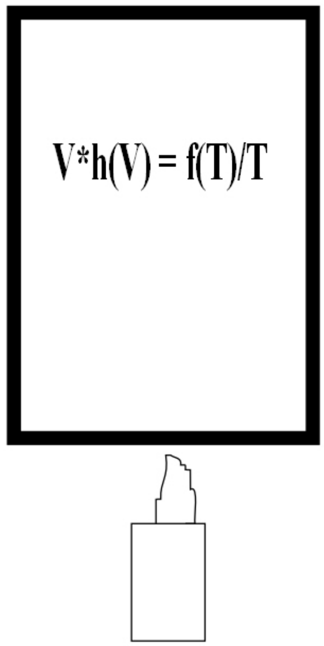
Note that the left hand side of this equation is a function of only the volume V, and the right hand side is a function of only the temperature T. And this equation must hold for any and all ideal gas behavior. Now suppose that I confine this gas inside a rigid steel container, so that the volume V CANNOT CHANGE. Then I gently heat the container with a candle. The temperature and pressure of the gas will certainly rise - BUT THE VOLUME V CANNOT CHANGE.



And yet the above equation must remain true!

Since the left hand side depends only on V, and cannot change value, the right hand side must also remain constant in value. This means that f(T) is such a function of T that, although T changes value, the quotient f(T)/T is a constant - under all conditions. Thus We see that f(T)/T = C, a constant (independent of P,V and T). 0r $f(T) = CT$. **C can depend only on the quantity of the gas.**

We have found the unknown function **f(T)** in Boyle's equation. Equation (1) thus becomes:

$$PV = CT$$, The general Ideal Gas equation.

Note that I used only equations (1) and (3) in this development. This was a free choice. You can produce the same result by similarly using ANY TWO of the three (Boyle, Charles, & Gay-Lussac) equations. DO IT!