A Generalized Ohm’s Law [*]

The usual microscopic (point) version of Ohm’s law $\vec{j} = s_0 \vec{E}$ applies to steady state DC currents and assumes the charge carriers have reached a steady terminal velocity. For transient situations, we will relax this assumption and examine the acceleration of the carriers. We apply Newton’s law to a charge carrier subject to an electric field and a coulomb frictional force:

1.) $m\dot{\vec{v}} = e\vec{E} - g\,\vec{v}$ (The usual Ohm’s law follows from setting the LHS = 0 and taking $\vec{v}$ to be a constant, terminal velocity.)

Multiply by the carrier charge e and sum over a unit volume:

2.) $$\sum em\dot{\vec{v}} = \sum e^2 \vec{E} - \sum eg\,\vec{v}$$

use: $\sum 1 = N$ = number of carriers per unit volume; and $\sum e\vec{v} = \vec{j}$

3.) $$m\frac{\partial \vec{j}}{\partial t} = Ne^2 \vec{E} - g\,\vec{j}$$

Under steady state DC conditions, when $\frac{\partial \vec{j}}{\partial t} = 0$, this must reduce to $\vec{j} = s_0 \vec{E}$.

Thus we can set: $s_0 = \frac{Ne^2}{g}$, the DC conductivity, and write:

4.) $$\frac{m}{g}\frac{\partial \vec{j}}{\partial t} + \vec{j} = s_0 \vec{E}$$ as a generalized “Ohm’s Law”.

If we define a time constant $t = \frac{m}{g}$ we have the version:

5.) $$t\,\frac{\partial \vec{j}}{\partial t} + \vec{j} = s_0 \vec{E}$$

The Time Behavior of Free Charge

We now wish to apply this to the time behavior of an unbalanced charge in the interior of a conductor. We seek a generalization of the usual relaxation equation $\frac{\partial r}{\partial t} + \frac{s_0}{e} r = 0$. We here assume medium properties (σ_0 , ε , γ) to be constant in time and space:

Taking the divergence of **5.)** and using $\nabla \cdot \vec{j} = -\frac{\partial r}{\partial t}$ and $\nabla \cdot \vec{E} = \frac{r}{e}$ yields

6.) $$t \frac{\partial^2 r}{\partial t^2} + \frac{\partial r}{\partial t} + \frac{s_0}{e} r = 0 \; .$$

(Note that for very small τ [a poor conductor] we could drop the first term and get the usual relaxation equation with a time constant ε/σ_0.)

Defining the "plasma frequency" $w_p^2 = \frac{s_0}{e\,t} = \frac{N e^2}{e\,m}$, we may write

7.) $$\frac{\partial^2 r}{\partial t^2} + \frac{1}{t}\frac{\partial r}{\partial t} + w_p^2 r = 0$$

This is a damped free oscillation at the "plasma frequency" ω_p with a damping time constant τ. Note that **7.)** is analogous to the equations describing free running damped harmonic oscillators and RLC circuits.

For copper: ω_p =1.6 E16 sec^{-1} , and τ = 2.4 E-14 sec.

* This discussion relies heavily on R. Becker and F. Sauter, **Electromagnetic Fields and Interactions**, Vol 1, pg 237-239, Blaisdell, 1964.