The qVxB force on a current carrying wire

The DC motor capitalizes on the qVxB force. But this force can do no work, since it is always perpendicular to v!! Where does the motor work come from? Of course (?) the work is done by the emf which maintains the current. But just how does this happen?

Here is my appreciation of the interaction of a B field with an emf driven current in a wire:

General notation: [x] specifies that x is a vector. Let [i], [j] and [k] be the unit x,y,z vectors : to the right, up the page, and out of the page, respectively. The wire lies along the x axis with the current consisting of positive carriers moving to the right with a drift velocity [v] = v[i]. The magnetic field is everywhere [B]= -B[k]:into the page.

The qVxB force gives the carriers an additional velocity component u[j], up the page. Thus, the (positive) carriers in the wire have a resultant velocity [w] = v[i] + u[j], with components to the right, and up the page. This resultant velocity is in the first quadrant of the xy plane making an angle TH with the y axis, where Tan(TH)=v/u. Note that TH is also the angle between the total magnetic force [w]x[B] and the negative x axis -[i].

The rate at which the magnetic force does work on a charge carrier is:

P = q*([w]x[B]) DOT [w] = q*([w]x[B]) DOT (v[i] + u[j])

Performing the DOT product: P = q* | [w]x[B] | * (-v*Cos(TH) + u*Sin(TH))

This is ostensibly zero, because v/u = tan(TH), by construction.

The magnetic force does no net work, but acts as a "go-between" to enable the external emf agent to do the work. The first term is power taken from the external agent; the second term is power given to the current carriers. One might say that the magnetic field delivers energy to the [j] motion of the carriers, but it gets that energy from the [i] motion of the carriers - which energy ultimately comes from whatever emf is driving the [i] current. Perhaps the situation is best summarized by saying that the qVxB force merely turns the carrier drift velocity through 90 degrees, while the emf restores the original drift velocity component - only this second step requires work.

I think I need to point out the important difference between the motion which I have just described of the carriers of an emf-driven current in a wire in the presence of a B field, and the circular "cyclotron" motion of a charged particle injected into a B field. The circular cyclotron motion results because the only force on the particle is the qVxB force, which cannot change the particle's speed; it can only turn it into a circular path, with its previous speed preserved. In the case of the current carrying wire, an emf maintains the (average) drift velocity component along the wire direction "no matter what". Since the qVxB force acts in a direction perpendicular to the velocity the overall kinematic effect is only to add a new perpendicular component to the carrier's velocity, because the emf continually "restores" the drift velocity component along the wire, re-plenishing the kinetic energy of this drift motion which has gone into the new perpendicular motion. The particle (average) motion is not a circle, but a straight line making an angle PHI with the wire direction, where tan(PHI) = u/v (the ratio of the new perpendicular velocity component to the emf-maintained drift velocity component). As already implied, this model speaks only of a carrier's average motion, abstracting from its erratic "thermal" excursions.

For completeness, it may be noted that the mechanism by which the emf drives the current is the ELECTRIC field of source charges which have accumulated at conductor surfaces and at internal locations of spatial variation of conductivity.