An alternate display of the world line in special relativity

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I. INTRODUCTION

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In teaching special relativity the plot of a particle's world line using the coordinates (x,ct) of the inertial laboratory frame is a very useful tool. However, there are some properties of this view which can leave a student with a confused feeling of incompleteness:

1) The particle world line (x = vt) is also the proper time axis of this particle, but the quantitative use of this axis to indicate proper time (cT) requires re-scaling by the use of a cumbersome hyperbolic calibration curve.

2) Nowhere in this view is the Lorentz invariant $S^2 = (ct)^2 - x^2$ overtly displayed. This trademark fact of special relativity cries out to be show-cased in any space-time display as an obvious "Pythagorean" right triangle.

A plot in the mixed space-time (cT,x) presents a view with the automatic display of much useful added information. Here T is the particle's proper time, and x is the particle's position as measured in the inertial laboratory frame. Note that the cT axis can also be labeled S, since in the particle's frame cT = S , the Lorentz invariant magnitude of the position four vector.

II. CONSTANT VELOCITY KINEMATICS

To implement this view we will translate the world line (x = vt) into the mixed (S,x) space-time. We simply substitute t = x/v into the Lorentz invariant $S^2 = (ct)^2 – x^2$ to yield:

$$S(x) = x\sqrt{\left(\frac{c}{v}\right)^2 - 1} \qquad (1)$$

This world line is plotted in fig. 1, using the mixed (S,x) space-time. Note that distance measured along this world line is equal to ct, where t is the inertial laboratory time. For each event the relation $S^2 = (ct)^2 – x^2$ is explicitly displayed with an obvious right triangle. The same is true for this relation applied to the incremental coordinates connecting any two events in this plot. For each event of the particle's existence this view makes explicit the values of S, cT, x and ct, in evident scales. Note also that the entire first quadrant of this plot is used for the world lines of particles with velocities ranging from 0 to c. In this mixed space-time view the world line of a photon is coincident with the x axis, illustrating that in the impossible photon frame, everything "happens" at once.

Note that a grid of constant laboratory times (ct = constant) can be easily added to these plots. They would be a family of concentric circular arcs, centered on the origin, and with radii equal to ct - with the same scale as x and cT..

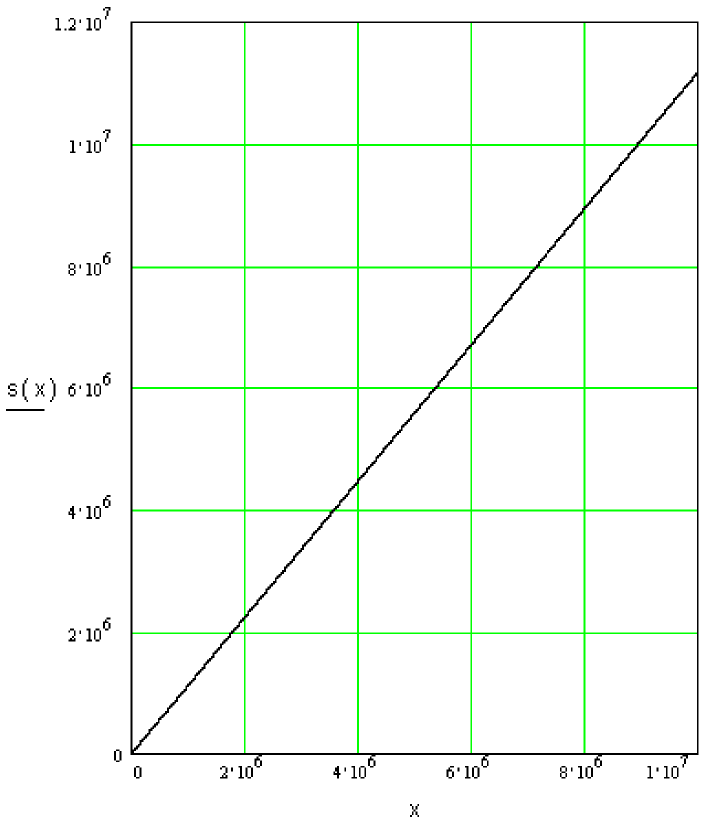

Fig. 1 A plot of equation (1), the world line x=vt translated into (S,x) space-time, using $v=2x10^8$ and $c=3x10^8$. Since $(ct)^2 = x^2 + S^2$, distance measured along the world line is equal to ct, where t is the time as measured in the inertial laboratory frame. Note also that S = cT, where T is the particle's proper time and x is the particle position as measured in the inertial laboratory frame.

III CONCLUSION

I do not propose that this (S,x) mixed space-time view should replace the (x,t) laboratory space-time view in our teaching. I propose that we continue to first present and explore the (x,t) view, and at an appropriate time (perhaps in response to confused student questions) introduce the (S,x) plot as an alternate view. The combination of both views should plant the seeds of a more complete and intuitive visualization of special relativistic kinematics.